

Third Semester B.E. Degree Examination, July/August 2004

EC / TE / ML / IT / BM / EE

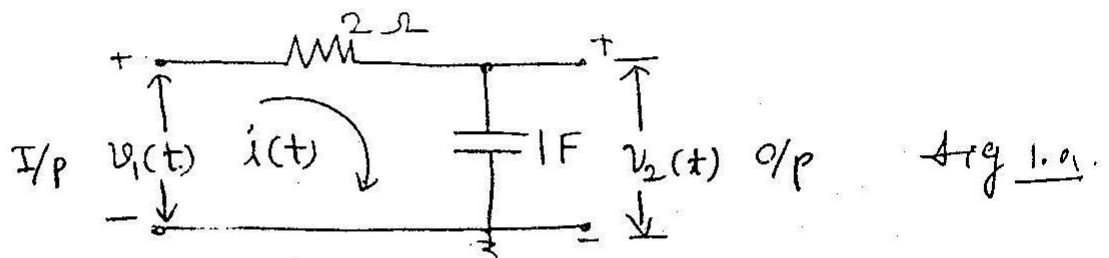
Signals and Systems

Time: 3 hrs.]

[Max.Marks : 100

Note: Answer any FIVE full questions.

1. (a) A continuous, causal Linear Time Invariant System is shown in Fig 1.a.



Determine the unit impulse and step response of this system. Plot the response. Also verify whether the system is causal and stable. (10 Marks)

- (b) Find
- $x(t) * y(t)$
- for the signals shown and also sketch the convolved signal

$$\text{i) } x(t) = \delta(t) - 2\delta(t-1) + 8\delta(t-2)$$

$$y(t) = 2, \quad -1 \leq t \leq 1$$

$$\text{ii) } x(t) = 2, \quad \text{for } -1 \leq t \leq 1$$

$$y(t) = t, \quad -2 \leq t \leq 2$$

(10 Marks)

2. (a) A discrete LTI system is characterized by the following difference equation

$$y(n) - y(n-1) - 2y(n-2) = x(n]$$

with $x(n) = 6u(n)$ and initial conditions
 $y(-1) = -1, y(-2) = 4$

- i) Find the zero-input response, zero-state response, and total response.
 ii) How does the total response change if $y(-1) = -1, y(-2) = 4$ as given, but $x(n) = 12u(n)$
 iii) How does the total response change if $x(n) = 6u(n)$ as given, but $y(-1) = -2$ and $y(-2) = 8$. (10 Marks)

- (b) State and prove frequency convolution and modulation property of Fourier transform of a CT signal. (10 Marks)

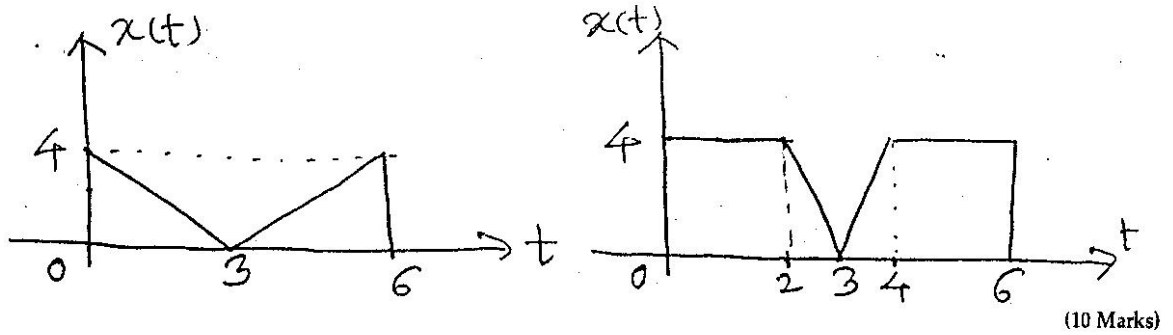
3. (a) i) Consider a linear shift - invariant system with unit - sample response
- $h(n) = \alpha^n u(n)$
- , where
- α
- is real and
- $0 < \alpha < 1$
- . If the input is
- $x(n) = \beta^n u(n)$
- for
- $0 < |\beta| < 1$
- , determine the output
- $y(n)$
- in the form
- $y(n) = (K_1 \alpha^n + K_2 \beta^n) u(n)$
- by explicitly evaluating the convolution sum.

- ii) By explicitly evaluating the transforms
- $x(e^{j\omega})$
- ,
- $H(e^{j\omega})$
- and
- $y(e^{j\omega})$
- corresponding to
- $x(n)$
- ,
- $h(n)$
- and
- $y(n)$
- specified in part (i), show that

$$y(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega})$$

(10 Marks)

(b) Find the Fourier transform of the following signals shown in fig (3b) showing uproperties of the Fourier transform.



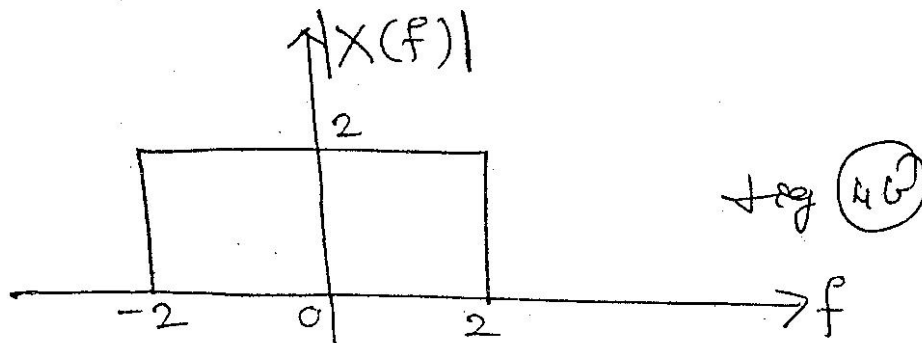
4. (a) An LTI system is described by

$$H(f) = \frac{4}{2 + j2\pi f}$$

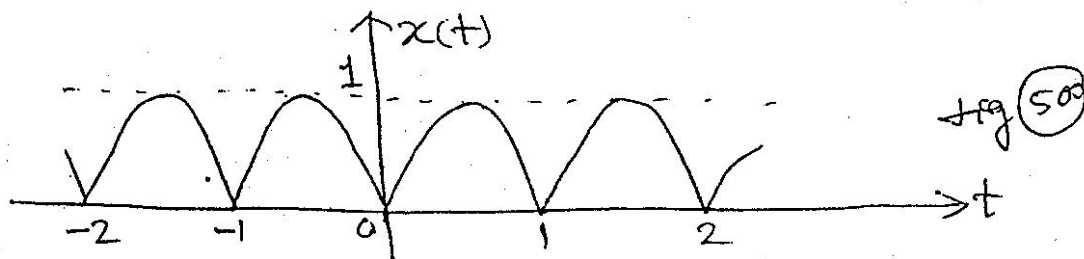
Find its response $y(t)$ if the input is $x(t) = u(t)$.

(10 Marks)

(b) State and prove Parseval's theorem for CT signals. Using this theorem determine the range of frequencies $(-f_1, f_1)$ where 50% of the signals energy lies. The spectrum of the signal is shown in fig.4.b.



5. (a) Determine the exponential Fourier series representation for the full rectified sinewave shown in Fig 5. (a). Also plot the line spectrum.



(b) Show that the Fourier transform of a train of impulses of unit height, separated by T secs, is also a train of impulses of height $\omega_0 = \frac{2\pi}{T}$, separated by $\omega_0 = \frac{2\pi}{T}$.

(10 Marks)

6. (a) State and prove the sampling theorem for low pass signals. Give the significance of this theorem.

(10 Marks)

- (b) A 100Hz sinusoid $x(t)$ is sampled at 240 Hz. Has aliasing occurred? How many full periods of $x(t)$ are required to obtain one period of the sampled signal? (4 Marks)
- (c) A 100Hz sinusoid is sampled at rates of 140Hz, 90Hz and 35Hz. In each case, has aliasing occurred, and if so, what is the aliased frequency. If the original signal has the form $x(t) = \cos(200\pi t + \theta)$, write the expressions for the aliased signal $x_a(t)$. (6 Marks)
7. (a) State and prove time reversal and time convolution property of Z-transform. (8 Marks)
- (b) Find the Z-transform of the following including R.O.C.
- $x(n) = -\left(\frac{1}{2}\right)^n u(-n-1)$
 - $x(n) = \alpha^{|n|}$, $-\infty < |n| < \infty$
 - $x(n) = Ar^n \cos(\omega_0 n + \phi) u(n)$, $0 < r < 1$. (12 Marks)
8. (a) Suppose $x(z)$ is given by $x(z) = \frac{z(z^2 - 4z + 5)}{(z-3)(z-2)(z-1)}$
Find $x(n)$ for the following ROCS (12 Marks)
- $2 < |z| < 3$
 - $|z| > 3$
 - $|z| < 1$.
- (b) Solve the following linear constant coefficient difference equation using Z-transform method (8 Marks)
- $$y(n) - \frac{3}{2}y(n-1) + \frac{1}{2}y(n-2) = \left(\frac{1}{4}\right)^n u(n)$$
- with initial conditions $y(-1) = 4$ & $y(-2) = 10$.